Information theory II

Fisica dell’Energia - a.a. 2020/2021

Transfer of information

Communication

- Communication is the **transfer of information** from one place to another.
- This should be done
  - as **efficiently** as possible
  - with as much **fidelity/reliability** as possible
  - as **securely** as possible
- Communication System: Components/subsystems act together to accomplish information transfer/exchange

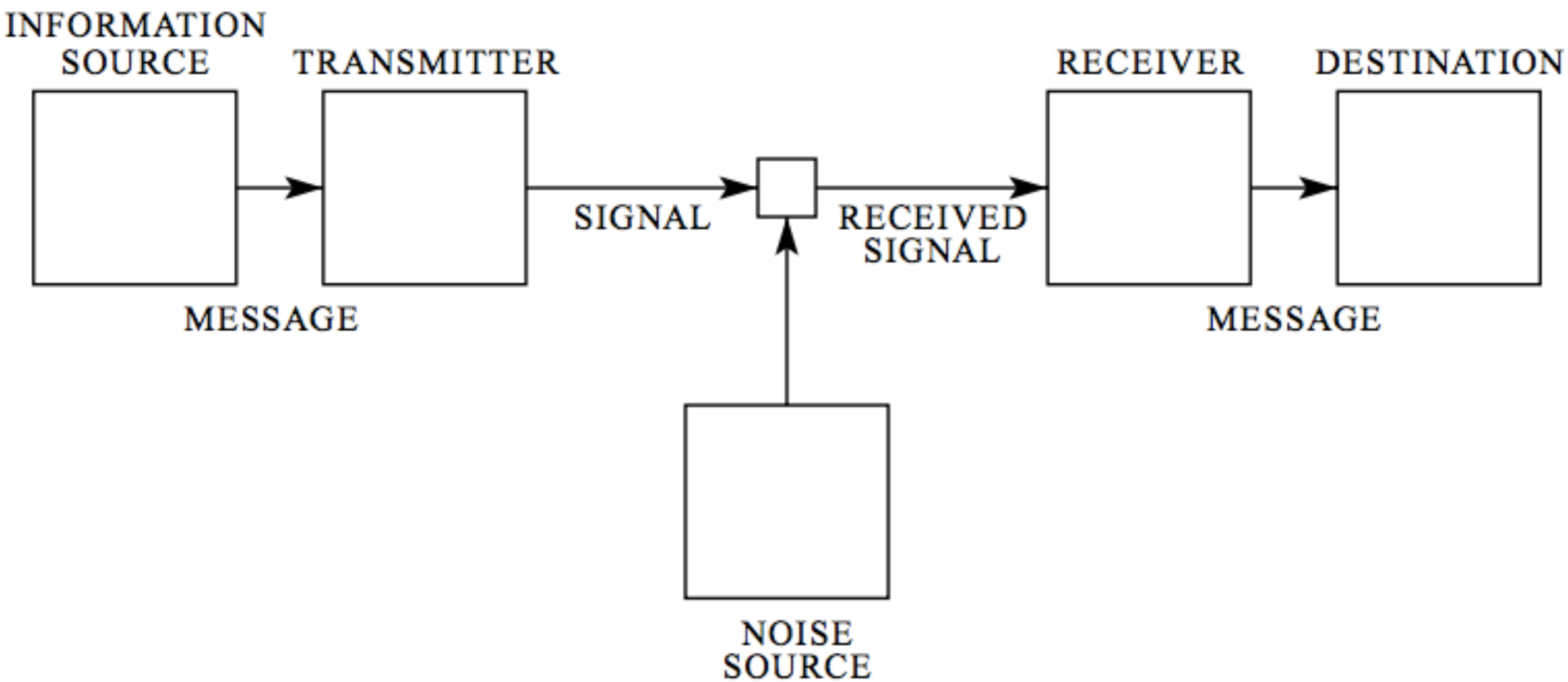
Communication

- **Verbal** Communication
  - Spoken communication
  - Languages and dialects
- **Written** Communication
  - Symbols, hieroglyphics, and drawings

Communication

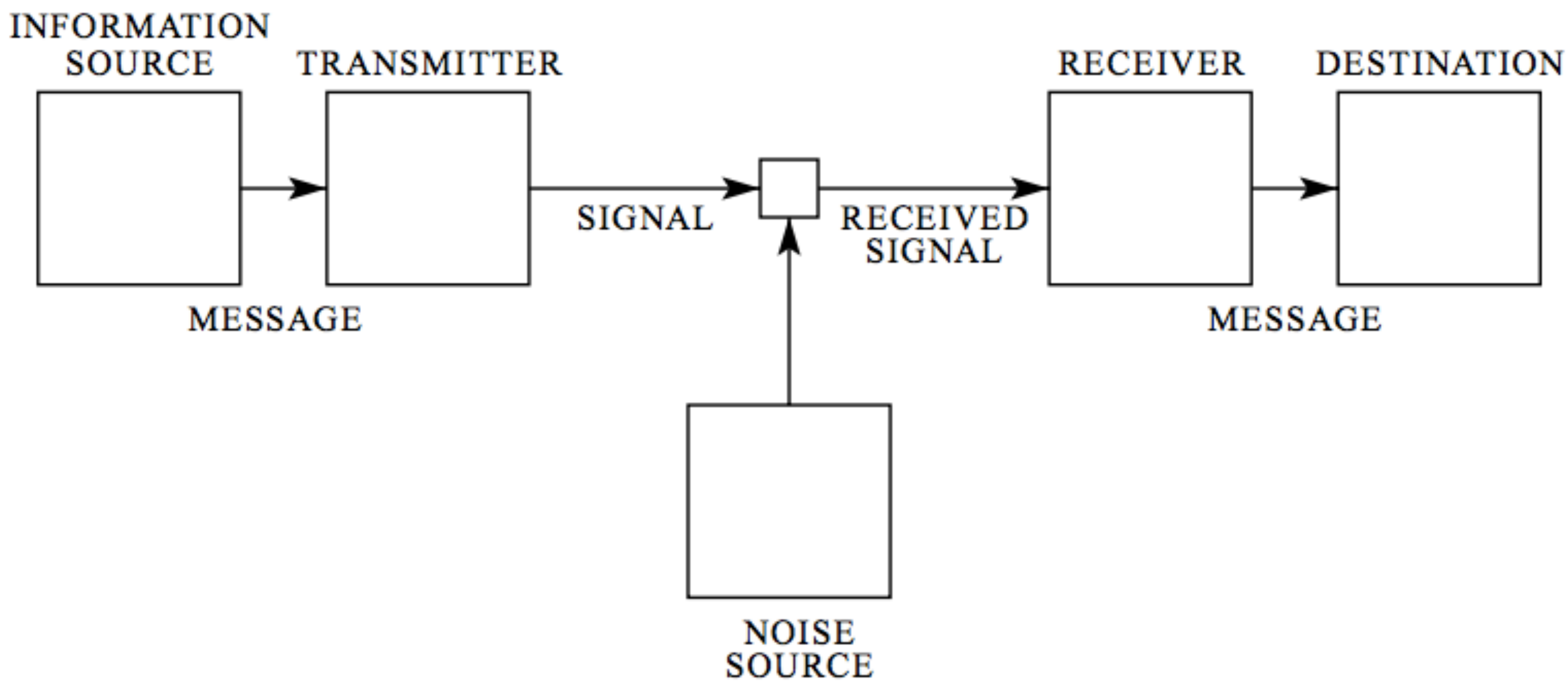
- Smoke signals, telegraph, telephone…
- 1895: invention of the radio by Marconi
- 1901: trans-atlantic communication
- …
- nowadays: everything communicate, Internet of things (IoT)

Communication system



A Mathematical Theory of Communication - By C. E. SHANNON

Communication system



- The message **produced by a source** must be converted by a transducer to a form suitable for the particular type of communication system.
- Example: In electrical communications, speech waves are converted by a microphone to voltage variation.

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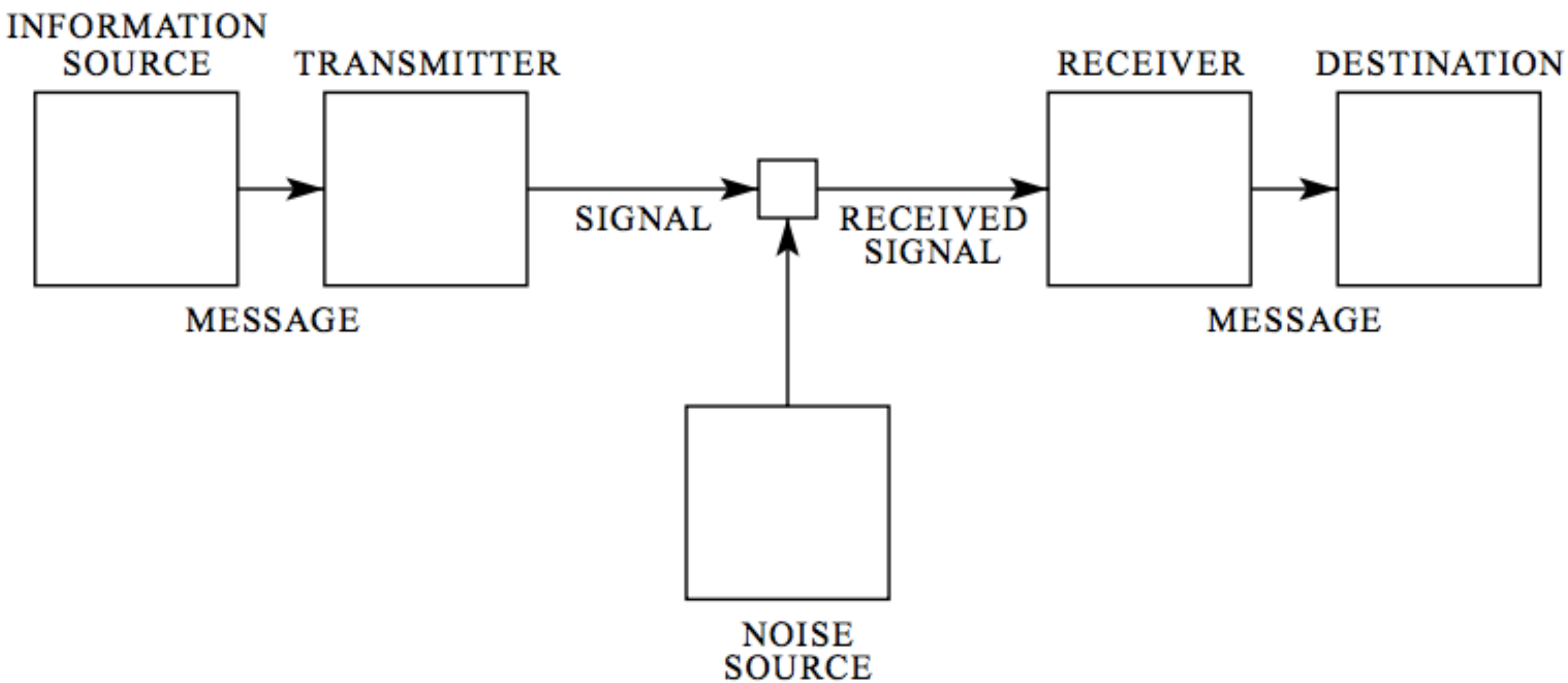
Communication system



- The **transmitter** processes the input signal to produce a signal suits to the characteristics of the transmission channel.
- Signal processing for transmission almost always involves modulation and may also include coding. In addition to modulation, other functions performed by the transmitter are amplification, filtering and coupling the modulated signal to the channel.

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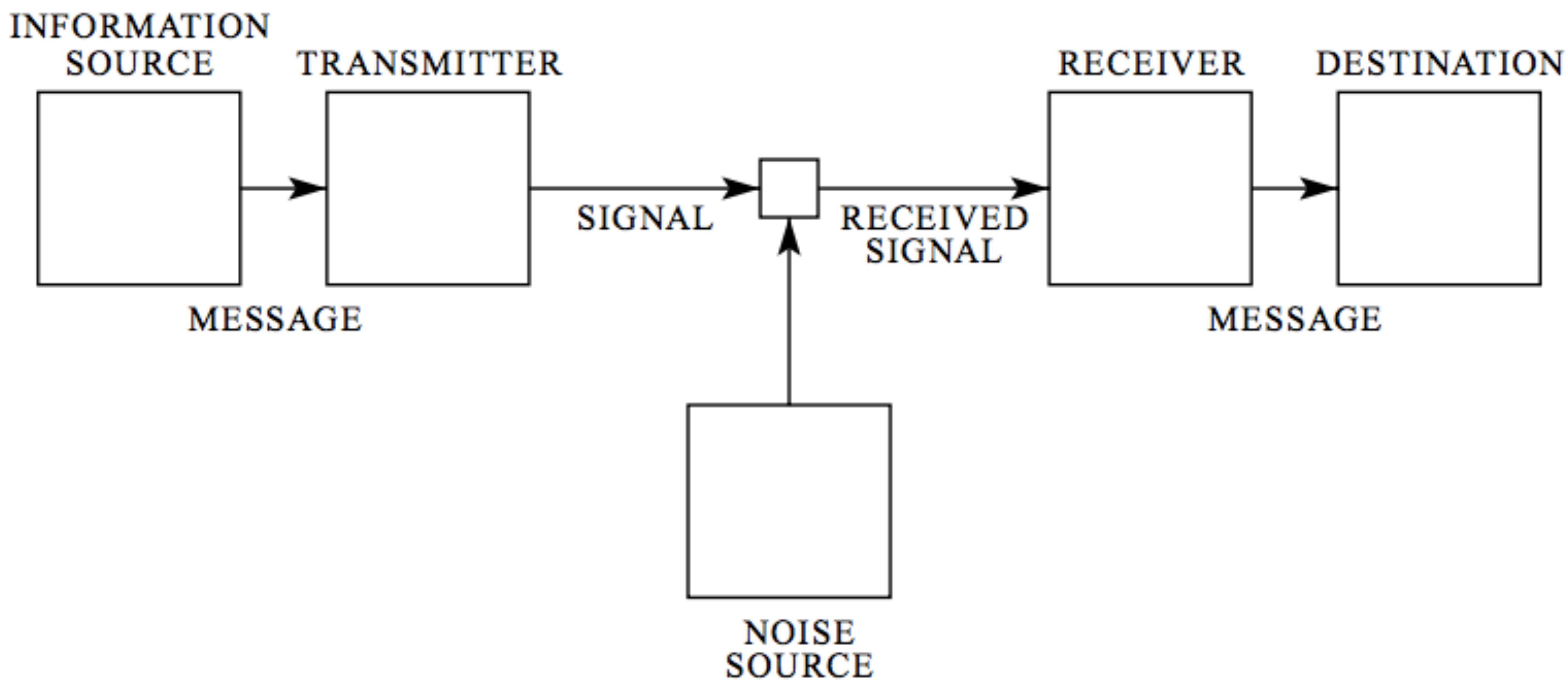

- The **receiver**'s function is to extract the desired signal from the received signal at the channel output and to convert it to a form suitable for the output transducer.

- Other functions performed by the receiver: amplification (the received signal may be extremely weak), demodulation and filtering.

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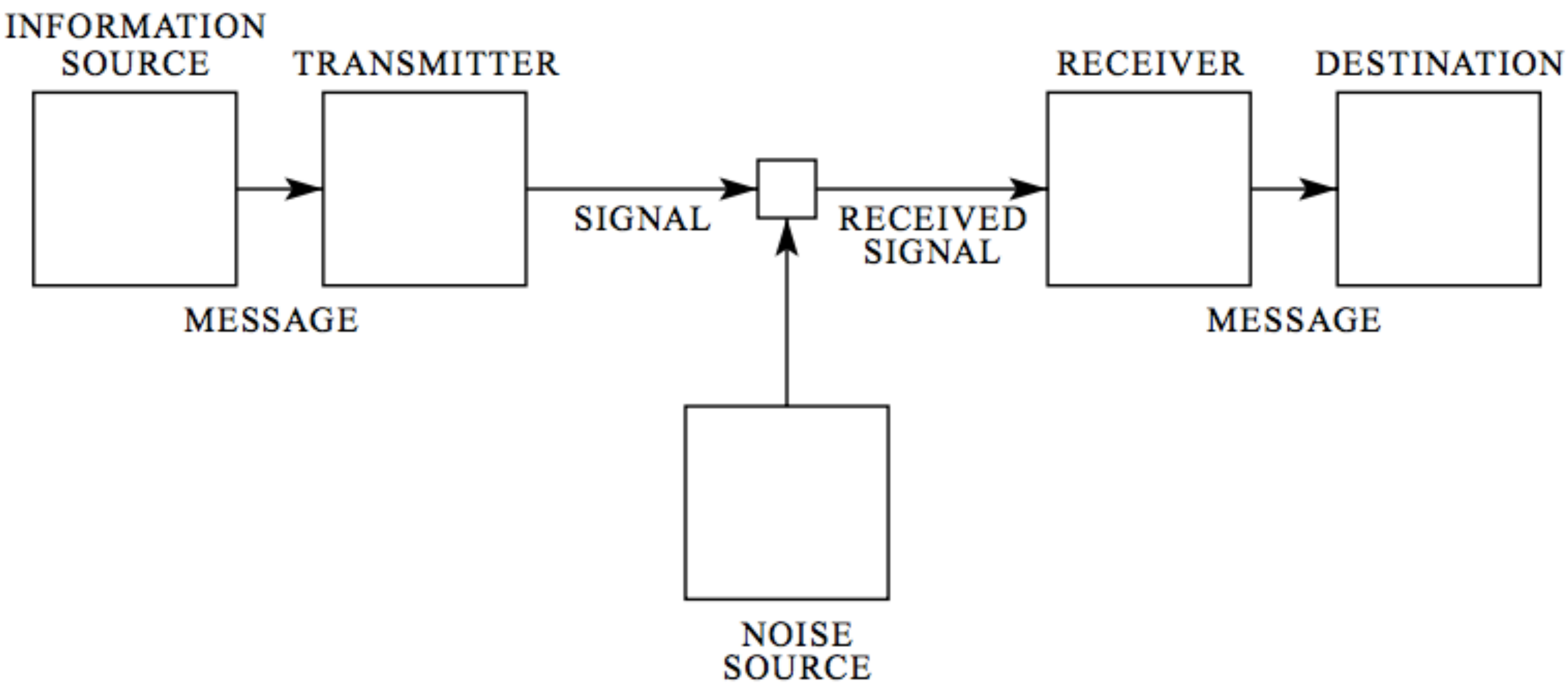

- The **output** transducer converts the electric signal at its input into the form desired by the system user.

- Example: Loudspeaker, personal computer (PC), tape recorders.

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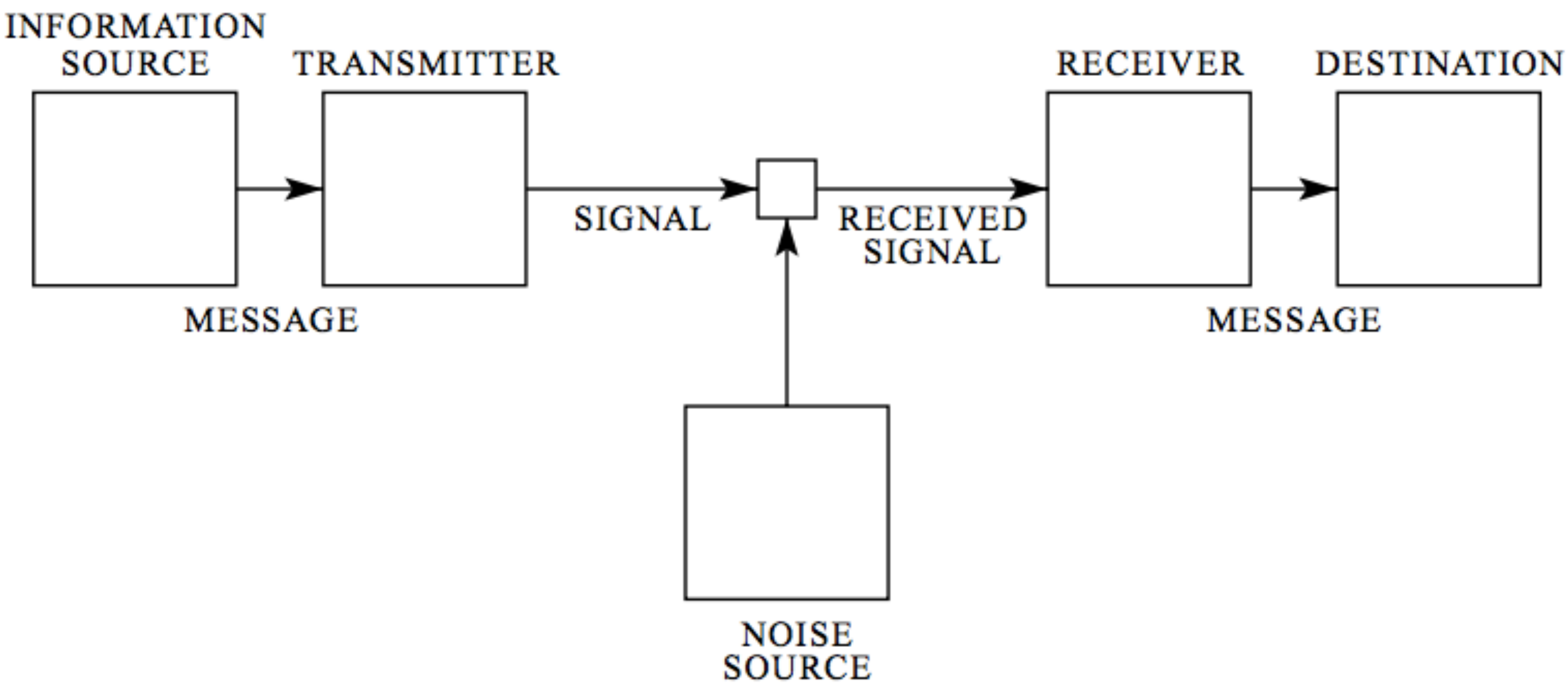
Communication system



- The **channel** can have different forms: the atmosphere (or free space), coaxial cable, fiber optic, waveguide, etc…

- The signal undergoes some amount of degradation from **noise**, **interference** and **distortion**

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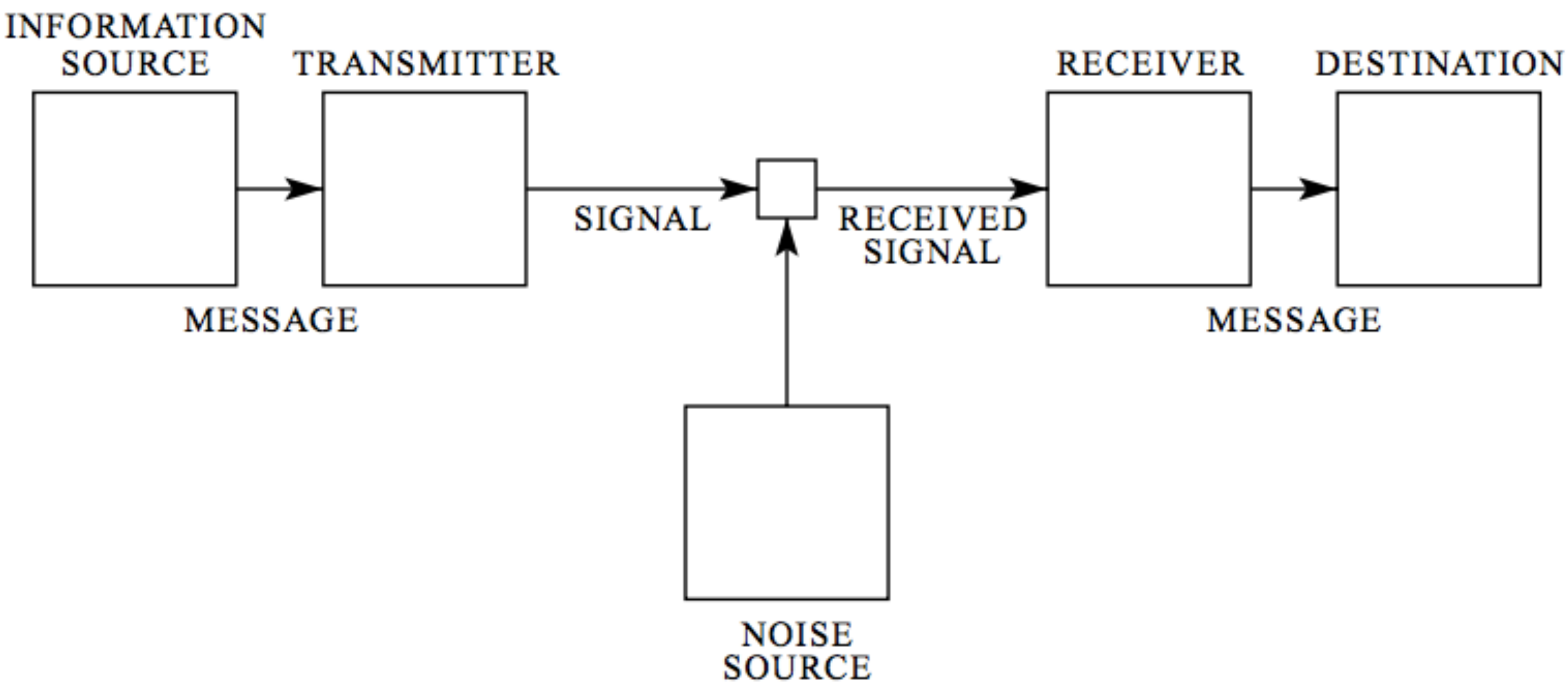
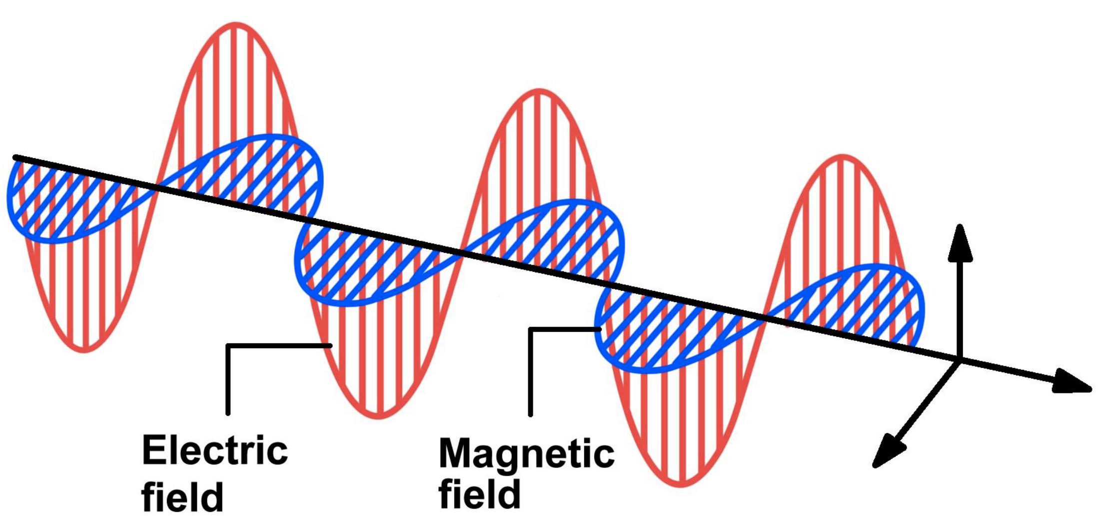
Communication system





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Communication system

Wave length	**Frequency Designations**	**Transmission Media**	**Propagation Modes**	**Representative Applications**	**Frequency**
1 cm	Extra High Frequency (EHF)	Wave guide		Satellite, Microwave relay, Earth-satellite radar.	100 GHz
10 cm	Super High Frequency (SHF)		Line-of-sight radio		10 GHz
1 m	Ultra High Frequency (UHF)			Wireless comm. service, Cellular, pagers, UHF TV	1 GHz
10m	Very High Frequency (VHF)	Coaxial Cable	Sky wave radio	Mobile, Aeronautical, VHF TV and FM, mobile radio	100 MHz
100m	High Frequency (HF)			Amateur radio, Civil Defense	10 MHz
1 km	Medium High Frequency (MF)		Ground wave radio	AM broadcasting	1 MHz
10 km	Low Frequency (LF)	Wire pairs		Aeronautical, Submarine cable, Navigation, Transoceanic radio	100 kHz
100km	Very Low Frequency (VLF)				10 kHz

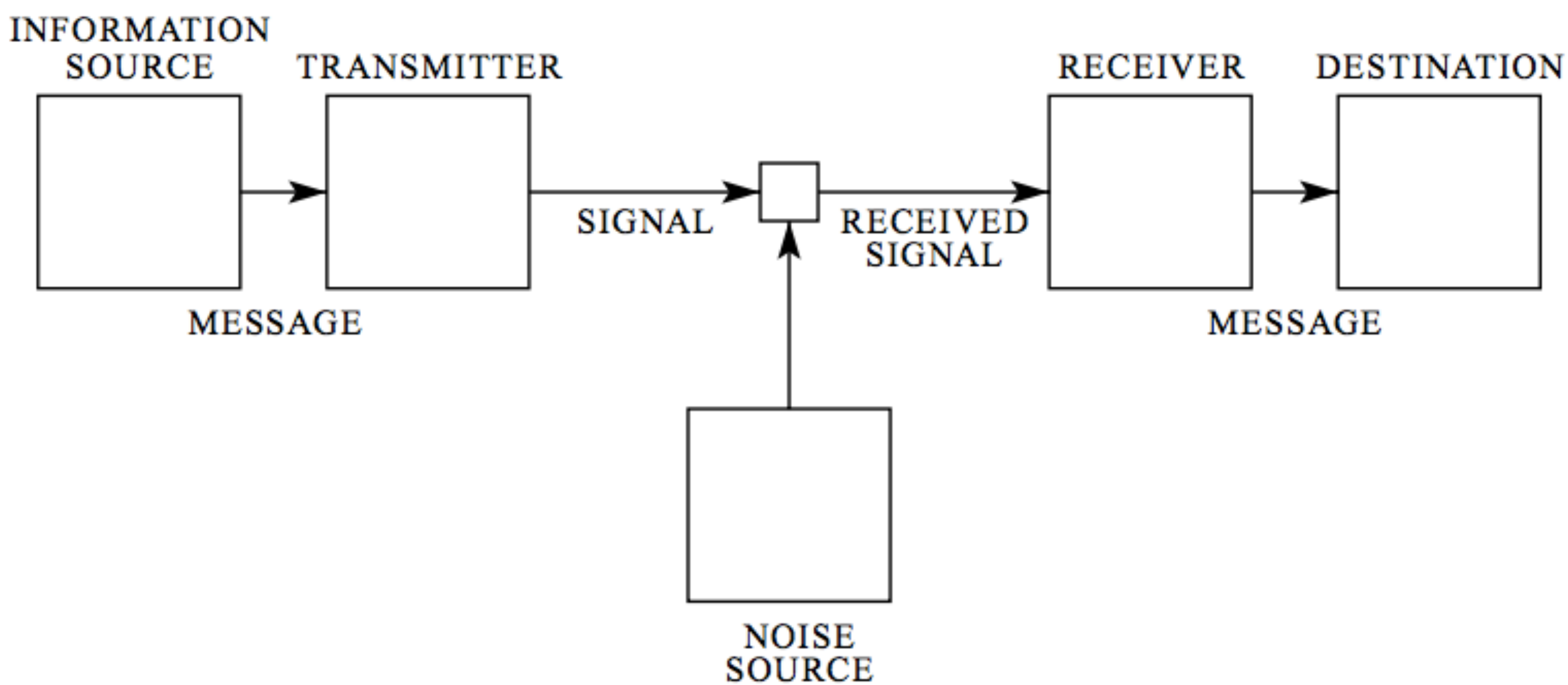
Signal

- The information to be transmitted can be **encoded** modulating amplitude (AM) or frequency (FM) of a signal

- According to Fourier analysis, any composite signal is a **combination** of simple sine waves with different **frequencies**, **amplitudes**, and **phases**

- The information **transmission rate** is **limited** by the transmitter, the medium and the receiver

Communication system



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Shannon’s channel capacity

- Shannon’s Channel Coding Theorem states that if the information rate R (bits/s) is equal to or less than the channel capacity, C, (i.e. $R < C$) then there is, in principle, a coding technique which enables transmission over the noisy channel with no errors.

- The inverse of this is that if $R > C$, then the probability of error is close to 1 for every symbol.

- The channel capacity is defined as: the maximum rate of reliable (error-free) information transmission through the channel.

Harry Nyquist

- Johnson-Nyquist noise
- Telegraphy
- Facsimile (Fax)
- Television

Nyquist formula

- Assume a channel is noise free.
- Nyquist formulation: if the rate of signal transmission is 2B, then a signal with frequencies no greater than B is sufficient to carry the signal rate.
    - given bandwidth B, highest signal rate is 2B.
- Given binary signal (two voltage levels), the maximum data rate supported by B Hz is 2B bps.

Nyquist formula

- Signals with more than two levels can be used, i.e., each signal element can represent more than one bit.
    - E.g., if a signal has 4 different levels, then a signal can be used to represents two bits: 00, 01, 10, 11
- With multilevel signalling, the Nyquist formula becomes:
    - $C = 2B \log_2 M$
    - M is the number of discrete signal levels, B is the given bandwidth, C is the channel capacity in bps.

Nyquist formula

$$C = 2B \log_2 M$$

- How large can M be?
    - The receiver must distinguish one of M possible signal elements.
    - Noise and other impairments on the transmission line will limit the practical value of M.

Nyquist formula

- Channel capacity is concerned with the information handling capacity of a given channel.
- It is affected by:
  - The attenuation of a channel which varies with frequency as well as channel length.
  - The noise induced into the channel which increases with distance.
  - Non-linear effects such as clipping on the signal.
- Some of the effects may change with time e.g. the frequency response of a copper cable changes with temperature and age.

Transmission Impairments

- With any communications system, the signal that is received may differ from the signal that is transmitted, due to various transmission impairments.
- Consequences:
  - For analog signals: degradation of signal quality
  - For digital signals: bit errors
- The most significant impairments include
  - Attenuation and attenuation distortion
  - Delay distortion
  - Noise

Attenuation

- Attenuation affects the propagation of waves and signals in electrical circuits, in optical fibres, as well as in air (radio waves).

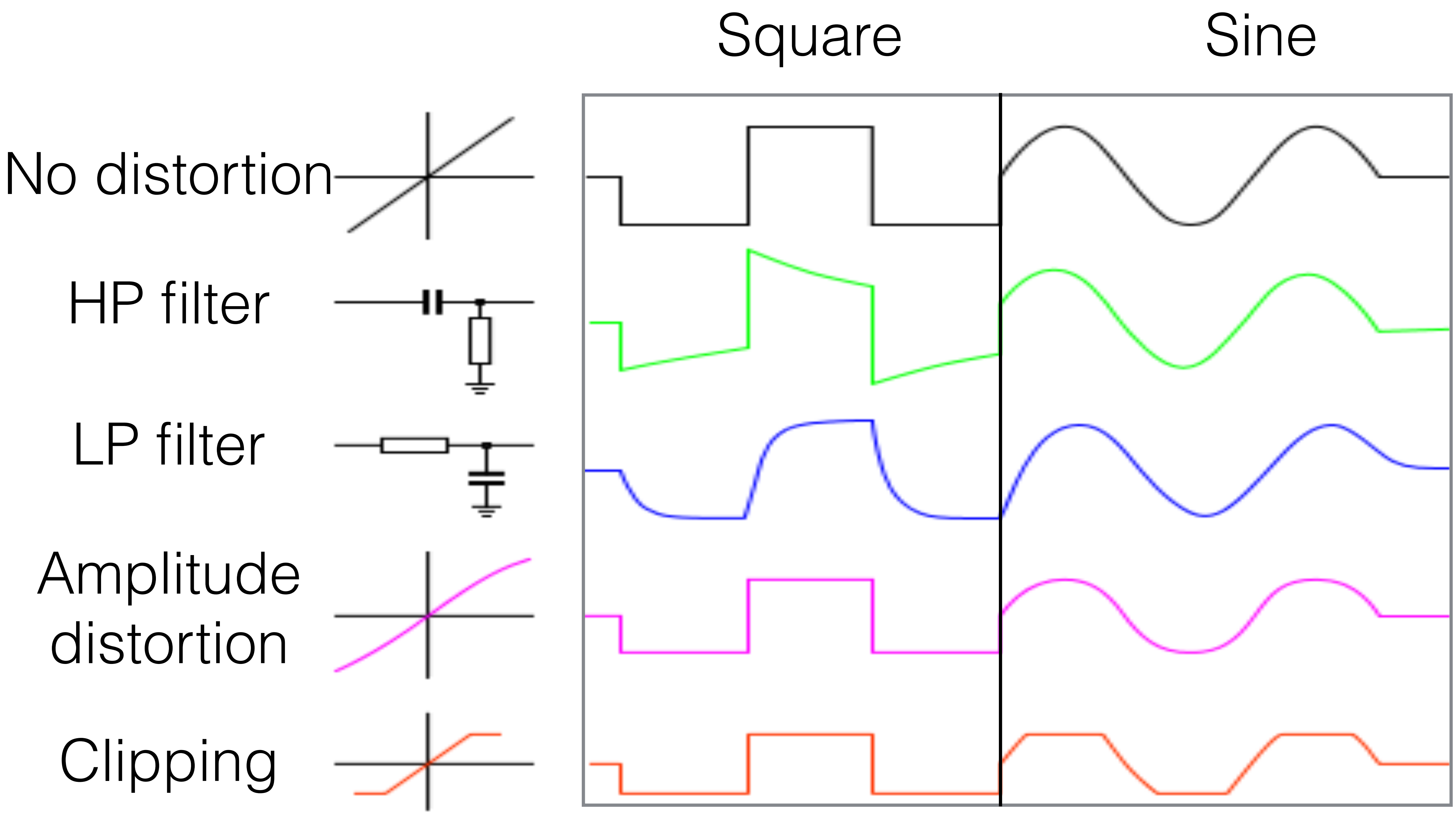
- Attenuation is reduction of signal strength during transmission. Attenuation is the opposite of amplification, and is normal when a signal is sent from one point to another. If the signal attenuates too much, it becomes unintelligible, which is why most networks require repeaters at regular intervals. Attenuation is measured in decibels.

Distortion

- Distortion is known as the alteration of the original signal. This may happen due to the properties of the medium. There are many types of distortion such as amplitude distortion, harmonic distortion, and phase distortion. For electromagnetic waves polarisation distortions also occurs. When the distortion occurs, shape of the waveform is changed.

- For example, amplitude distortion happens if all the parts of the signals are not equally amplified. This happens in wireless transmissions because the medium get changed by the time. The receivers should be able to identify these distortions.

Distortion


Square
Sine
No distortion
HP filter
LP filter
Amplitude distortion
Clipping

Difference between attenuation and distortion

- Although scaled down in amplitude, shape of waveform does not change in attenuation unlike in distortion.

- Removal of the effects of attenuation is easier than a removing the effects distortion.

- If the attenuation happens in different amounts for the different parts of the signal, it is a distortion.

Noise

- For any data transmission event, the received signal will consist of the transmitted signal, modified by the various distortions imposed by the transmission system, plus additional unwanted signals that are inserted somewhere between transmission and reception.

- The undesired signals are referred to as noise, which is the major limiting factor in communications system performance.

- Four main categories of noise:

  - Thermal noise

  - Intermodulation noise

  - Crosstalk

  - Impulse noise

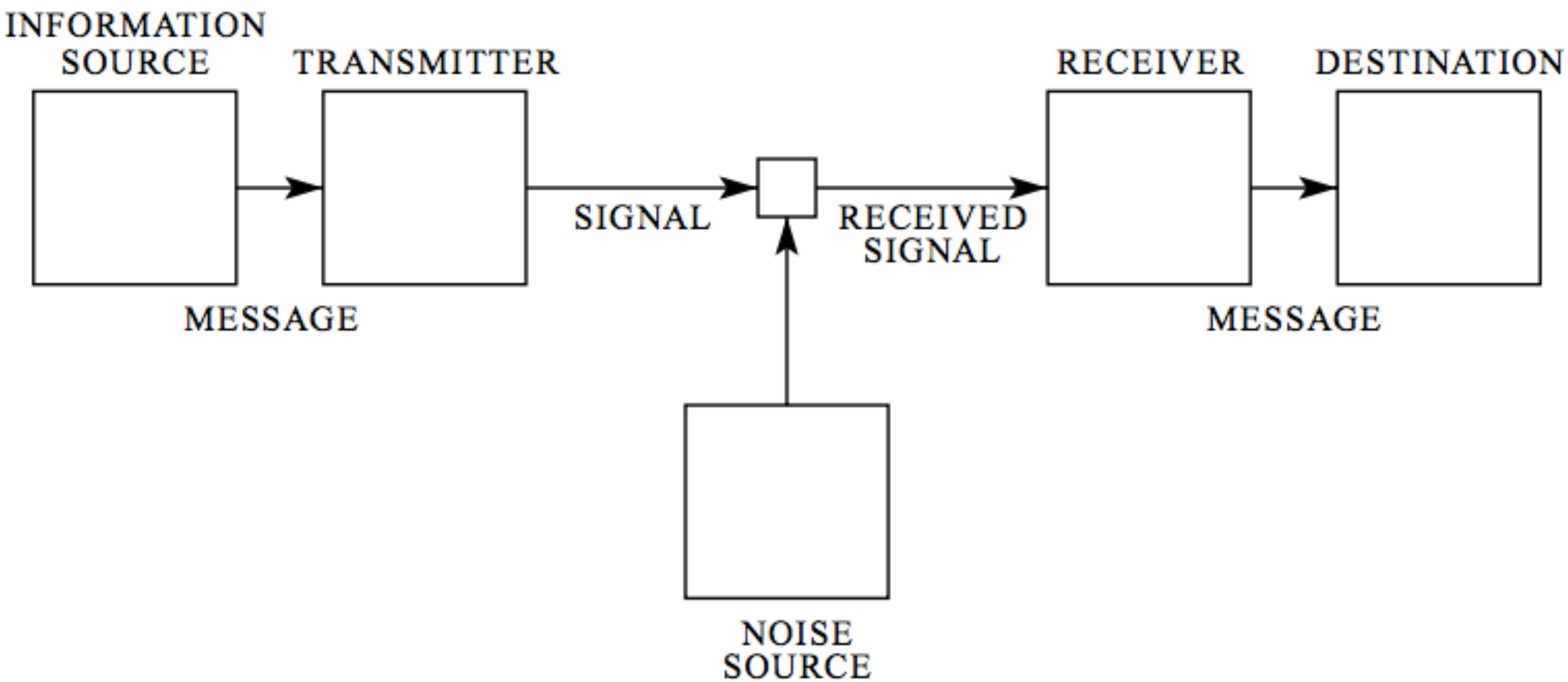
Noise

- **Thermal noise**
  - Due to thermal agitation of electrons
  - It is present in all electronic devices and transmission media, and is a function of temperature.
  - Cannot be eliminated, and therefore places an upper bound on communications system performance.
- **Intermodulation noise**
  - When signals at different frequencies share the same transmission medium, the result may be intermodulation noise.
  - Signals at a frequency that is the sum or difference of original frequencies or multiples of those frequencies will be produced.
  - E.g., the mixing of signals at f1 and f2 might produce energy at frequency f1 + f2. This derived signal could interfere with an intended signal at the frequency f1 + f2.

Noise

- **Crosstalk**
  - It is an unwanted coupling between signal paths. It can occur by electrical coupling between nearby twisted pairs.
  - Typically, crosstalk is of the same order of magnitude as thermal noise.
- **Impulse noise**
  - Impulse noise is non-continuous, consisting of irregular pulses or noise spikes of short duration and of relatively high amplitude.
  - It is generated from a variety of cause, e.g., external electromagnetic disturbances such as lightning.
  - It is generally only a minor annoyance for analog data.
  - But it is the primary source of error in digital data communication.

Communication system



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Shannon's channel capacity

- Shannon's Channel Capacity Theorem (or the Shannon-Hartley Theorem) states that:

$$C = B\log_2\left(1 + \frac{S}{N}\right) \quad bits/s$$

- where C is the channel capacity, B is the channel bandwidth in hertz, S is the signal power and N is the noise power.

- Note: S/N is the ratio watt/watt not dB.

Shannon's channel capacity

- The channel capacity, C, increases as the available bandwidth increases and as the signal to noise ratio increases (improves).
- This expression applies to information in any format and to both analogue and digital communications, but its application is most common in digital communications.
- The channel capacity theorem is one of the most important results of information theory. In a single formula it highlights the interplay between 3 key system parameters:
  - channel bandwidth,
  - average transmitted signal power,
  - noise power at the channel output.

Shannon’s channel capacity

- For a given average transmitted power S, a noise power N, and channel bandwidth, B, we can transmit information at the rate C bits/s with no error, by employing sufficiently complex coding systems. It is not possible to transmit at a rate higher than C bits/s by any coding system without a finite probability of error.

- Hence the channel capacity theorem defines the fundamental limit on the rate of error-free transmission for a power-limited, band-limited channel.

Capacity vs Bandwidth

$$C = B\log_2\left(1+\frac{S}{N}\right) \quad bits/s$$

- It appears from the expression that as the bandwidth increases the capacity should increase proportionately.
- But this does not happen, because increasing the bandwidth, B, also increases the noise power N = N_0B giving:

$$\begin{aligned} C &= B\log_2\left(1+\frac{S}{N}\right) \\ &= B\log_2\left(1+\frac{S}{N_0B}\right) \\ &= \frac{S}{N_0}\frac{N_0B}{S}\log_2\left(1+\frac{S}{N_0B}\right) \\ &= \frac{S}{N_0}\log_2\left(1+\frac{S}{N_0B}\right)^{\frac{N_0B}{S}} \end{aligned}$$

Capacity vs Bandwidth

- Consider the case where an infinite bandwidth is available. Increasing B to infinity means that S/N_0B tends to zero.
- Notice that $\lim_{x \to 0} (1+x)^{1/x} = e$
- This means that as the bandwidth goes to infinity S/N_0B goes to 0 and $(1+S/N_0B)^{N_0B/S}$ goes to e.
- The channel capacity therefore goes to :

$$\lim_{B \to \infty} C = \lim_{B \to \infty} \frac{S}{N_0} \log_2(1 + S/N_0B)^{N_0B/S} = \frac{S}{N_0} \log_2 e = 1.44 \frac{S}{N_0}$$

- So as the bandwidth goes to infinity the capacity goes to $1.44\ S/N_0$, i.e. it goes to a finite value and is not infinite.

Analog to digital

Harry Nyquist

- Determined that the **number of independent pulses** that could be put through a telegraph channel per unit time is **limited to twice the bandwidth** of the channel
- *Certain factors affecting telegraph speed* (1924)
- *Certain topics in Telegraph Transmission Theory* (1928)
- This rule is essentially a dual of what is now known as the **Nyquist–Shannon sampling theorem**

Nyquist–Shannon sampling theorem

- From **continuous** signal to **discrete** signal
- **Sampling** is the process of converting a signal into a numeric sequence
- Applies to signals whose Fourier transform are **zero outside of a finite region** of frequencies
- The fidelity of the result depends on the **sampling rate** of the original samples
- **No actual information is lost** during the sampling process

Nyquist–Shannon sampling theorem

- If a function *x(t)* contains no frequencies higher than ***B*** cps (Hz), it is **completely determined** by giving its ordinates at a series of points spaced ***1/(2B)*** seconds apart

  - A sufficient **sample-rate** is therefore *2B* samples/ second, or anything larger

  - For a given sample rate ***fs*** the bandlimit for **perfect reconstruction** is $B \leq fs/2$

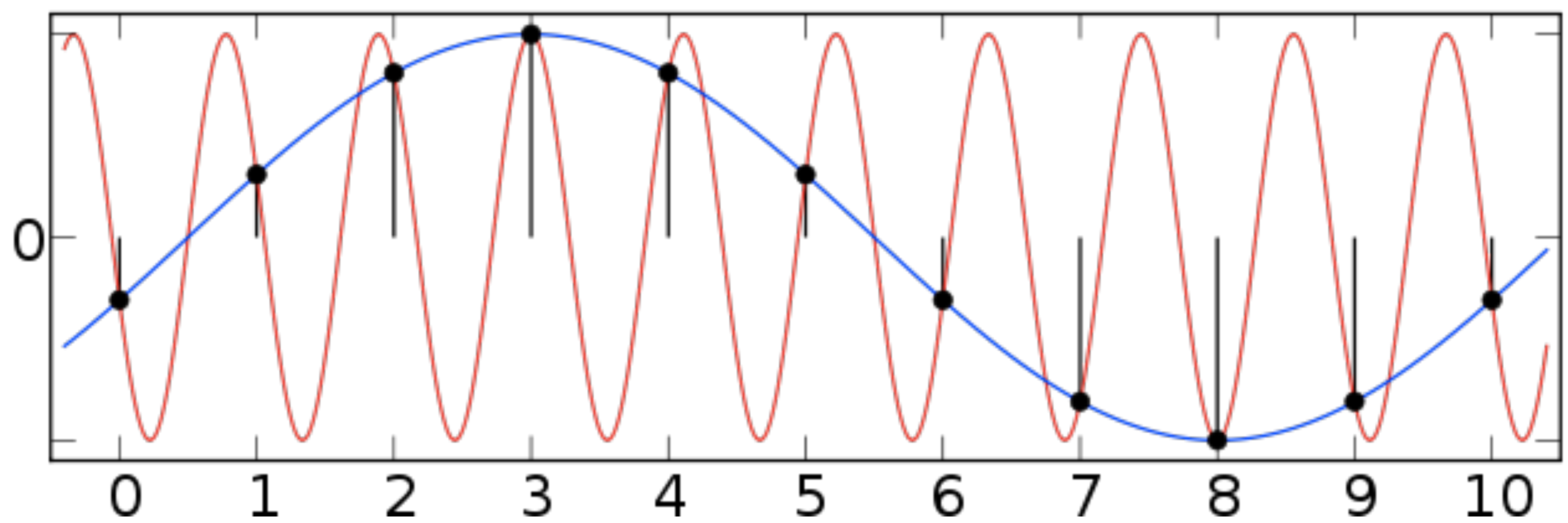
- *2B*: Nyquist rate

- *fs/2*: Nyquist frequency

Aliasing

https://www.youtube.com/watch?v=cMZJZGtWFCA

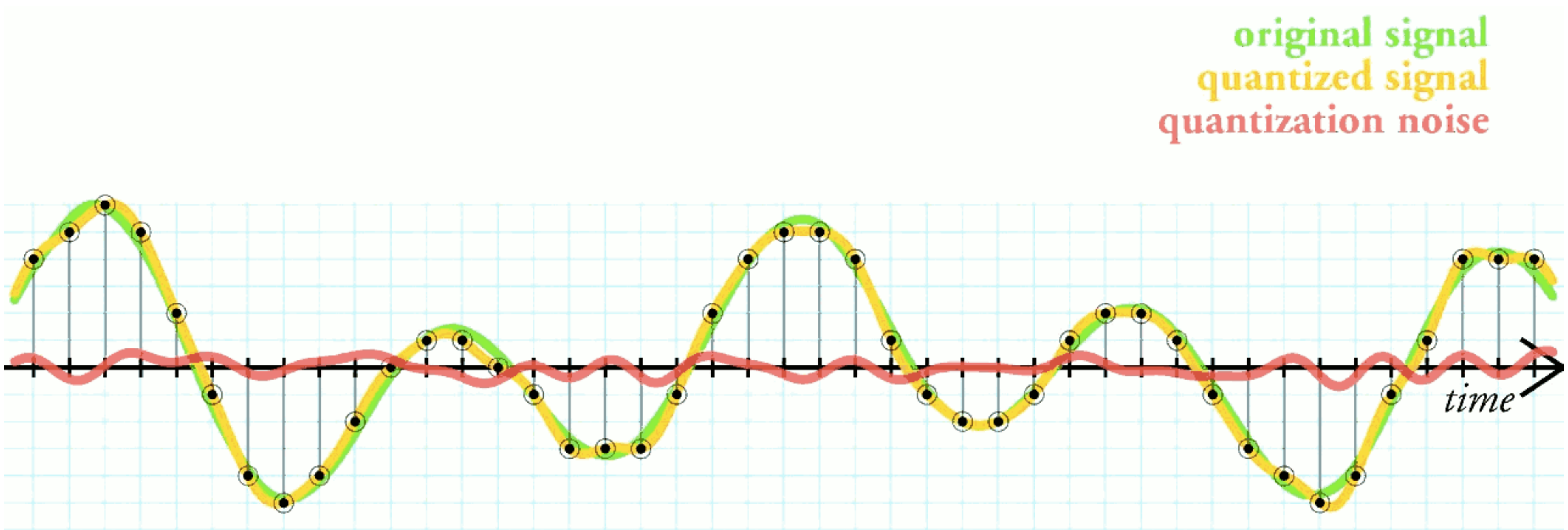
Aliasing



Two different sinusoids that fit the same set of samples.

Quantization noise

- Quantization, in mathematics and digital signal processing, is the process of mapping a large set of input values to a (countable) smaller set

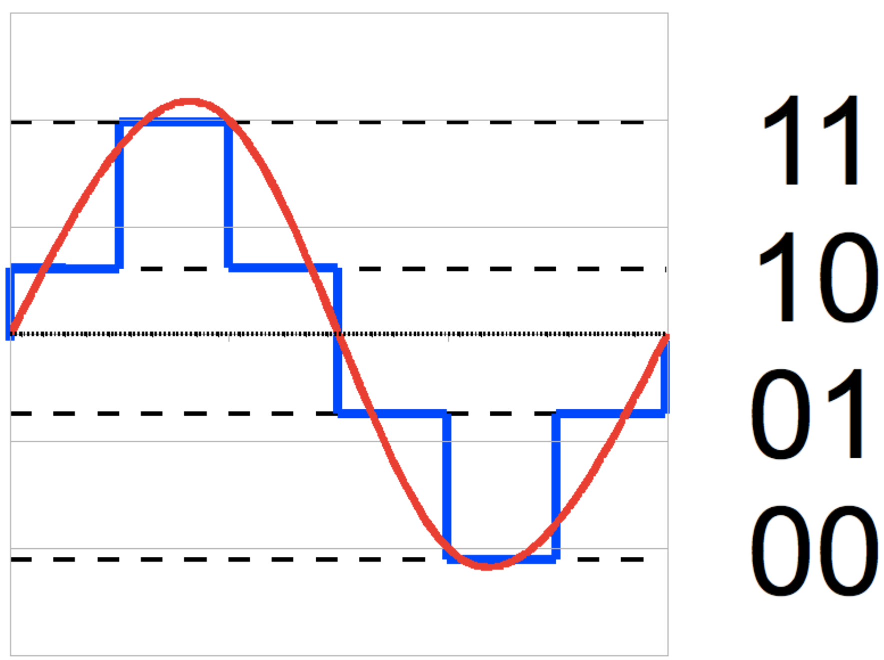


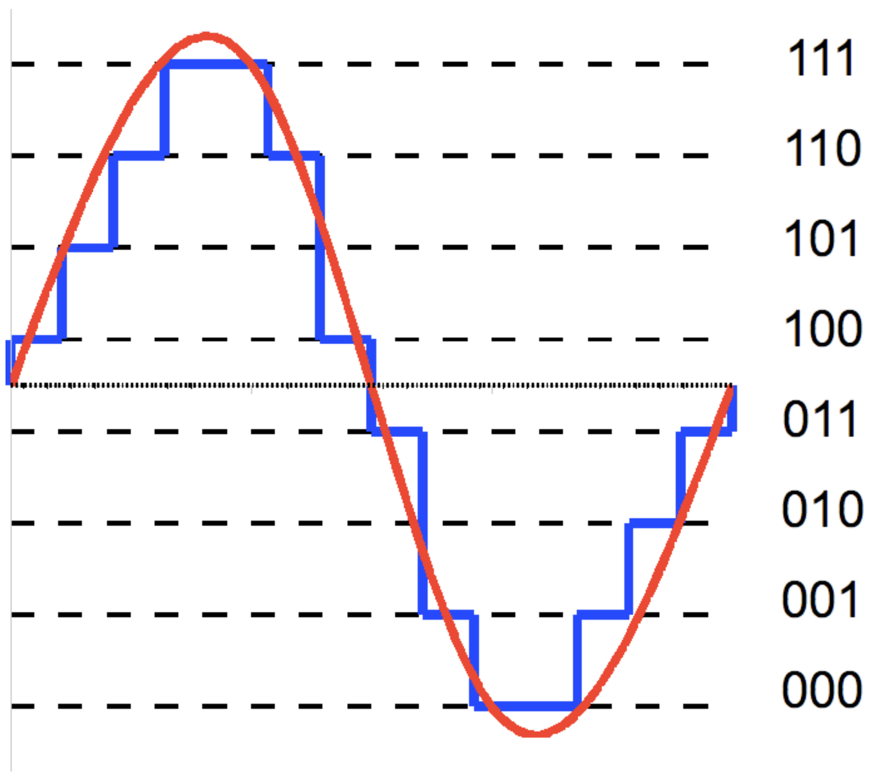
Quantization noise

- Quantization noise can’t be eliminated
- There is no Shannon-Nyquist analogous for quantization
- Quantization noise can be reduced:
  - increasing number of bit
  - increasing the sample rate (oversampling)
  - introducing additional noise

Quantization noise

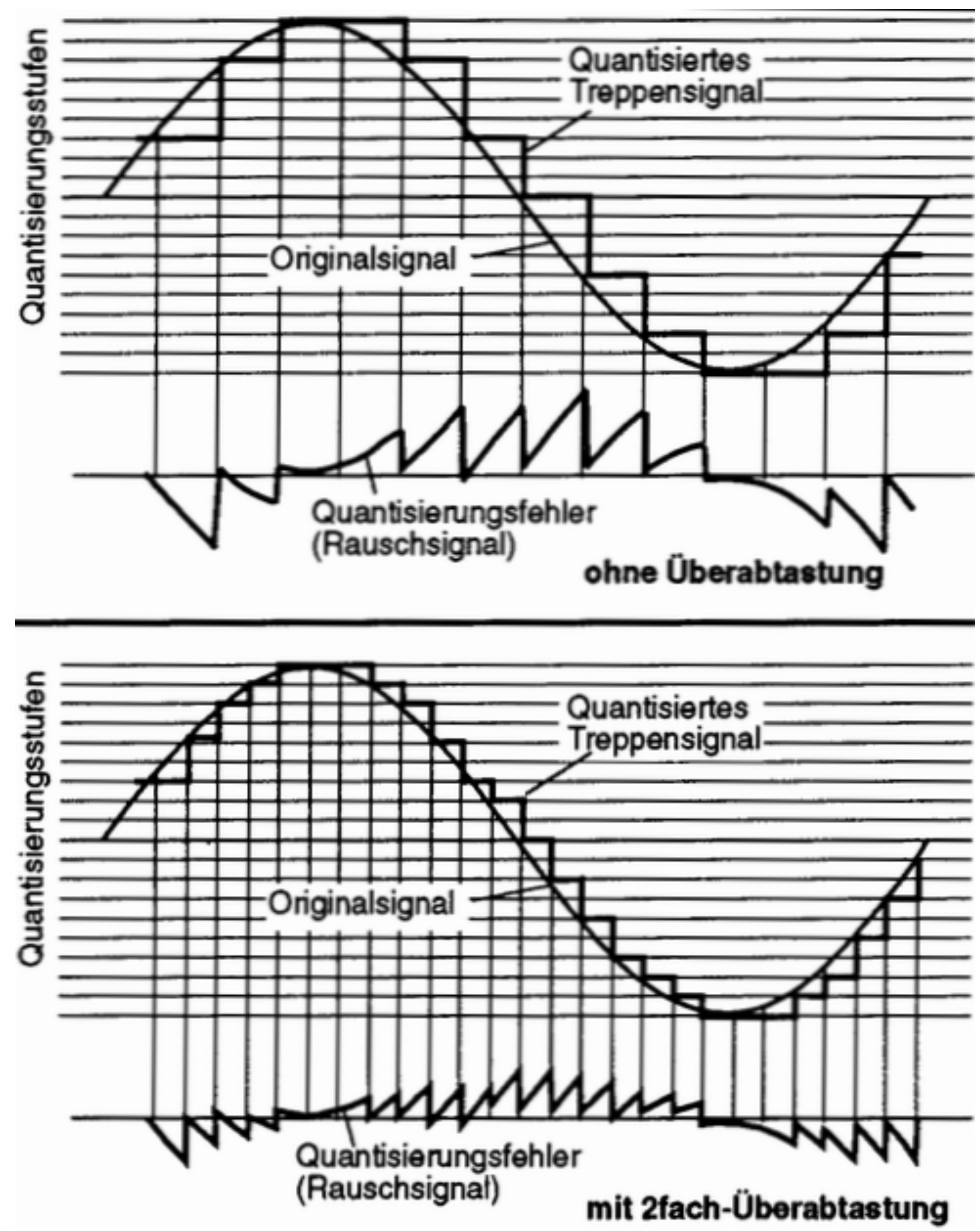
- Increasing number of bit





Quantization noise

- Increasing the sample rate (oversampling): the process of sampling a signal with a sampling frequency significantly higher than the Nyquist rate



Quantization noise

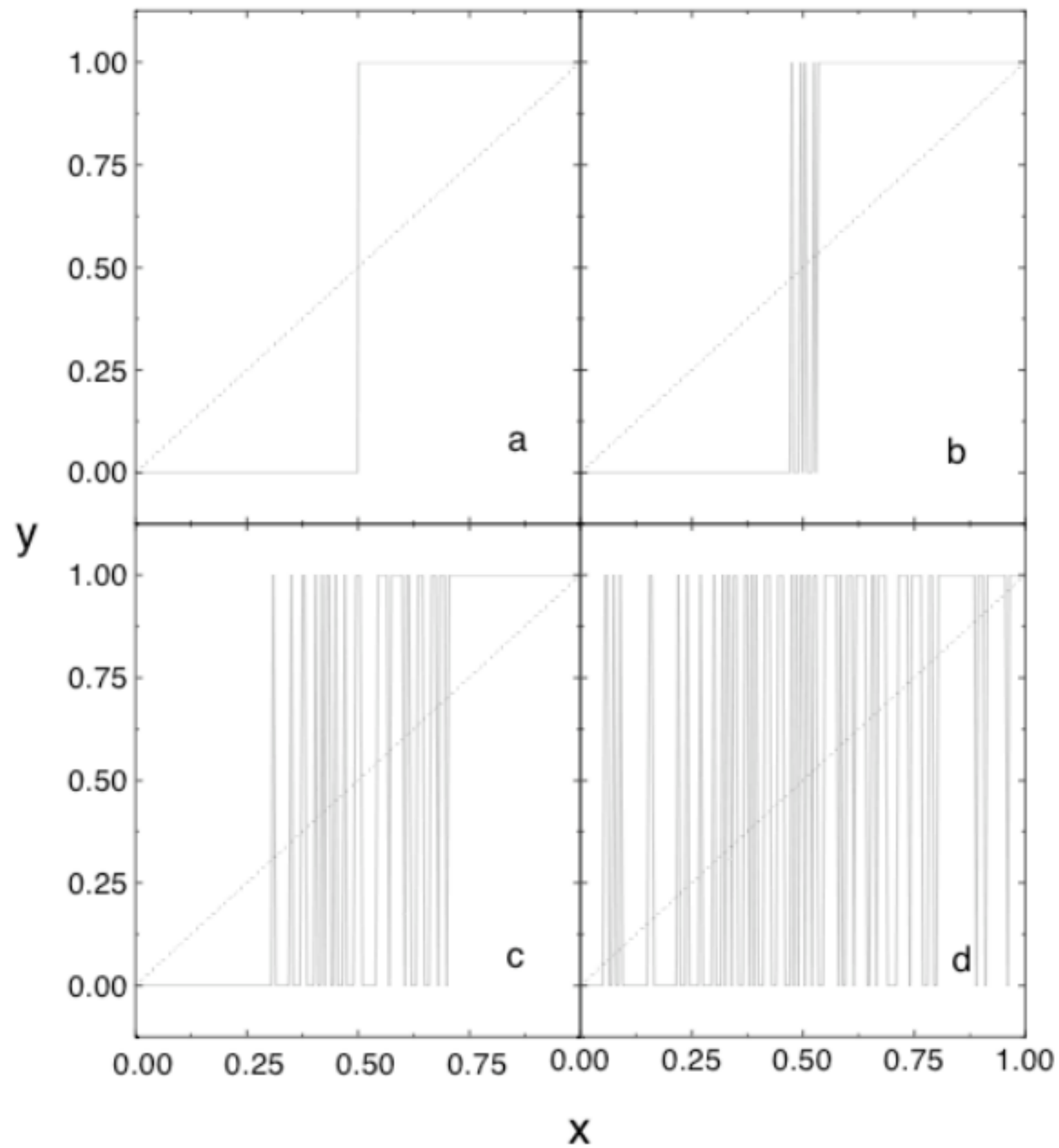
The Dithering effect:

…one of the earliest [applications] of dither came in World War II from Hrant H. Papazian. Airplane bombers used mechanical computers to perform navigation and bomb trajectory calculations. Curiously, these computers (boxes filled with hundreds of gears and cogs) performed more accurately when flying on board the aircraft, and less well on ground. Engineers realized that the vibration from the aircraft reduced the error from sticky moving parts. Instead of moving in short jerks, they moved more continuously. Small vibrating motors were built into the computers, and their vibration was called 'dither' from the Middle English verb 'didderen,' meaning 'to tremble.'

Ken Pohlmann, Principles of Digital Audio, 4th edition, page 46

Quantization noise

- The Dithering effect:



Quantization noise

- The Dithering effect: